Rate of Change

Math 130 - Essentials of Calculus

23 September 2019

Average Rate of Change

Definition

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If $y = f(x)$, then we say a corresponding change in y would be given by

$$\Delta y = f(x_2) - f(x_1).$$

Average Rate of Change

Definition (Average Rate of Change)

The average rate of change of y with respect to x over the interval $[x_1, x_2]$ is the difference quotient

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Example

Find the average rate of change of the function $f(x) = x^2 + 4x$ over the interval $[2, 5]$.

Average Rate of Change

Example

The table below shows the cumulative number of confirmed cases of H5N1 avian influenza worldwide on various dates in 2006, according to the World Health Organization:

Date	Cases
January 5	144
February 2	161
March 1	174
April 3	190
May 4	206
June 6	225

1. *Compute the average rate of change of the number of cases from March 1 to June 6 and interpret your result.*
2. *Find the average rate of change from April 3 to May 4.*

Now You Try It!

Example

The balance in an investment account t years after the account is opened is given by $B(t) = 9500(1.064^t)$. Compute the average rate of change for $2.5 \leq t \leq 4.5$ and interpret your result in this context.

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Solution

$$\frac{B(4.5) - B(2.5)}{4 - 5 - 2.5} \approx 732.72$$

Geometric Interpretation

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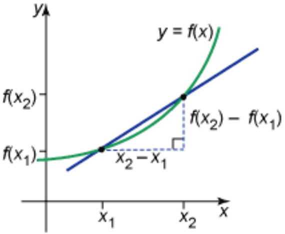
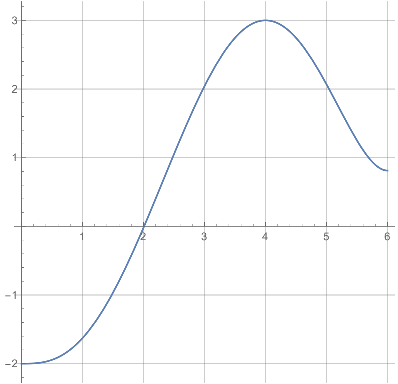
Hopefully the formation of the difference quotient reminds you of a slope. In fact, the average rate of change of the function $y = f(x)$ between x_1 and x_2 is just the slope of the secant line connecting the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ on the graph:



Average Rate of Change

Here is the graph of a function f



1. Sketch on the graph a secant line whose slope represents the average rate of change over $[2, 6]$, then compute this average rate of change.
2. Is the average rate of change of f over $[0, 3]$ positive or negative?
3. Which interval gives a larger average rate of change, $[1, 2]$ or $[3, 4]$?

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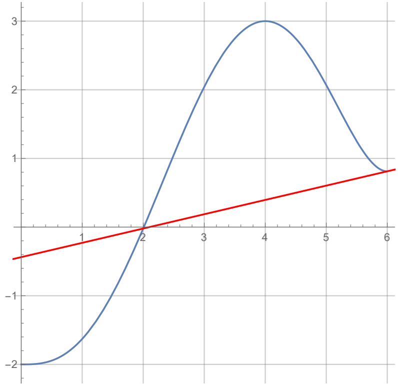

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Shrinking the Interval

Instead of searching for the average rate of change over an interval, we are often interested in the rate at which something is changing at a precise moment of time. We can try to approximate it by taking the average rate of change over smaller and smaller intervals and looking to see if there is some value which the averages are trending toward.

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Example

The height in feet of a baseball, t seconds after being thrown straight upward, is given by $h(t) = 36t - 16t^2$.

1. *Find the average speed of the ball for*
    1. $0 \leq t \leq 1$
    2. $0.5 \leq t \leq 1$
    3. $0.9 \leq t \leq 1$
    4. $0.99 \leq t \leq 1$
2. *Estimate the speed of the ball after 1 second.*

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The process we were taking in the last example is that of a *limit*. We are limiting, in that case, the time increment to 0.

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What is happening to the value of $f(x)$ as the value of x is getting closer to 1?

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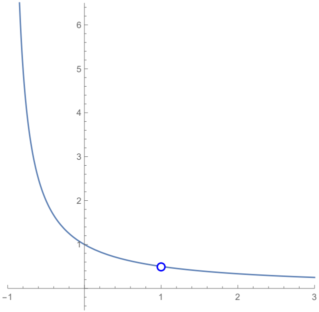

Concept of a Limit

What is happening to the value of $f(x)$ as the value of x is getting closer to 1? On the left side, values are approaching 1 from the left, and values are approaching 1 from the right in the second pair of columns.

x	$f(x)$	x	$f(x)$

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0.98	0.50505	1.02	0.49505
0.99	0.50251	1.01	0.49751

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0.99	0.50251	1.01	0.49751
0.995	0.50125	1.005	0.49875
0.999	0.50025	1.001	0.49975

So it appears the values are approaching 0.5. We say $\lim_{x\to 1} \dfrac{x-1}{x^2-1} = 0.5$.

Definition of a Limit

Definition

We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"
if the values of $f(x)$ approach L as the values of x approach a (but are not equal to a).